

Generic Construction of UC-Secure Oblivious Transfer

O. Blazy, C.Chevalier



- 1 Global Framework
- 2 Cryptographic Tools
- 3 1-out-of- t Oblivious Transfer
- 4 Instantiation
- 5 Conclusion

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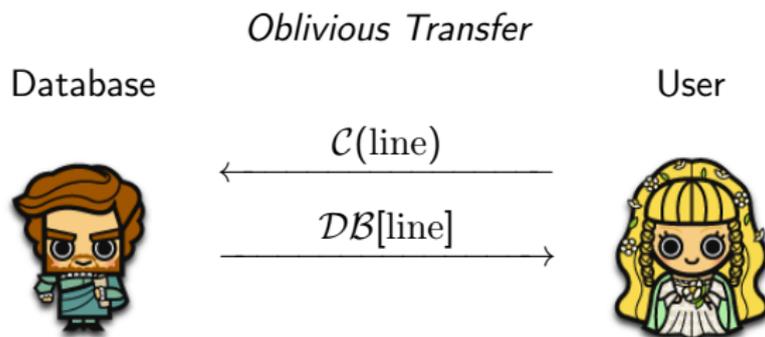
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 - Motivation
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Conditional Actions



- ↪ The User learns the value of line but nothing else.
- ↪ The Database learns nothing.

Semantic security

- Only the requested line should be learned by the User

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Oblivious

- The authority should not learn which line was requested

1 Global Framework

2 Cryptographic Tools

- Encryption Scheme
- Chameleon Hash Scheme
- Smooth Projective Hash Function

3 1-out-of- t Oblivious Transfer

4 Instantiation

5 Conclusion

Definition (Encryption Scheme)

$\mathcal{E} = (\text{Setup}, \text{KeyGen}, \text{Encrypt}, \text{Decrypt})$:

- $\text{Setup}(\mathcal{K})$: param;
- $\text{KeyGen}(\text{param})$: public *encryption* key pk , private *decryption* key dk ;
- $\text{Encrypt}(\text{pk}, m; r)$: ciphertext c on $m \in \mathcal{M}$ and pk ;
- $\text{Decrypt}(\text{dk}, c)$: decrypts c under dk .

Indistinguishability under Chosen Ciphertext Attack

Definition (Chameleon Hash Scheme)

CH = (Setup, KeyGen, CH, Coll):

- Setup(κ): param;
- KeyGen(param): outputs the chameleon hash key ck and the trapdoor tk ;
- CH($ck, m; r$): Picks r , and outputs the hash a ;
- Coll(ck, m, r, m', tk): Takes tk , (m, r) and m' , and outputs r' such that $CH(ck, m; r) = CH(ck, m'; r')$.

Extra Procedures (Verification)

- VKeyGen(ck): Outputs vk and vtk . \perp or public if publicly verifiable.
- Valid(ck, vk, m, a, d, vtk): Allows to check that d opens a to m .

Collision Resistance *

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- Setup(κ): param;
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Definition (Smooth Projective Hash Functions)

[CS02]

Let $\{H\}$ be a family of functions:

- X , domain of these functions
- L , subset (a language) of this domain

such that, for any point x in L , $H(x)$ can be computed by using

- either a *secret* hashing key hk : $H(x) = \text{Hash}_L(hk; x)$;
- or a *public* projected key hp : $H'(x) = \text{ProjHash}_L(hp; x, w)$

Public mapping $hk \mapsto hp = \text{ProjKG}_L(hk, x)$

Properties

For any $x \in X$, $H(x) = \text{Hash}_L(\text{hk}; x)$

For any $x \in L$, $H(x) = \text{ProjHash}_L(\text{hp}; x, w)$ w witness that $x \in L$

Smoothness

For any $x \notin L$, $H(x)$ and hp are independent

Pseudo-Randomness

For any $x \in L$, $H(x)$ is pseudo-random, without a witness w

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 - Definition
 - Our Generic Construction
 - Security
- 4 Instantiation
- 5 Conclusion

A user U wants to access a line ℓ in a database D composed of t of them:

- U learns nothing more than the value of the line ℓ
- D does not learn which line was accessed by U

Correctness: if U request a single line, he learns it

Security Notions

- Oblivious: D does not know learn which line was accessed ;
- Semantic Security: U does not learn any information about the other lines.

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Generic bit UC Commitment

- User picks a bit b , random r, d_{1-b}, \vec{s} , and computes $(a, d_b) = \text{CH}(\text{ck}, b; r)$
- He then computes $\mathcal{C} = \text{Encrypt}(d_0, d_1; \vec{s})$.

SPHF Compatibility

If the encryption is SPHF friendly, then one can build an SPHF on the language of valid encryption of a chameleon information.

$$\mathcal{L}_b = \{c \mid \exists d_{1-b}, s, \text{Valid}(\text{ck}, \text{vk}, b, a, d_b, \text{vtk}) \wedge c = \text{Encrypt}(d_0, d_1; s)\}$$

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Generic 1-out-of- t Oblivious Transfer

- User U picks ℓ :
For each bit, picks random $r_i, d_{1-\ell,i}$, and computes $(a_i, d_{\ell,i}) = \text{CH}(\text{ck}, \ell_i; r_i)$
He then computes $\mathcal{C} = \text{Encrypt}(\vec{d}; \vec{s})$ and sends \mathcal{C}, \vec{a} .
- For each line L_j , server S computes hk_j, hp_j , and $H_j = \text{Hash}_{\mathcal{L}_j}(\text{hk}_j, \mathcal{C})$,
 $M_j = H_j \oplus L_j$ and sends M_j, hp_j .
- For the line ℓ , user computes $H'_\ell = \text{ProjHash}_{\mathcal{L}_\ell}(\text{hp}_\ell, \mathcal{C}, \vec{s}_\ell)$, and then
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Security Properties

- ✓ Oblivious: IND-CCA security of the encryption scheme;
- ✓ Semantic Security: Smoothness of the SPHF / Collision Resistance of the Chameleon Hash
- ✓ UC simulation: Collision algorithm (Equivocation) of the Chameleon hash

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Chameleon Hash: Discrete Logarithm

[Ped91]

- $\text{KeyGen}(\mathcal{R})$: Outputs $\text{ck} = (g, h)$ $\text{tk} = \alpha = \log_g(h)$;
- $\text{VKeyGen}(\text{ck})$: Generates $\text{vk} = f$ and $\text{vtk} = \log_g(f)$
- $\text{CH}(\text{ck}, \text{vk}, m; r)$: $s \xleftarrow{\$} \mathbb{Z}_p$, and outputs $a = h^s g^m$, $d = f^s$.
- $\text{Coll}(m, s, m', \text{tk})$: Outputs $s' = s + (m - m')/\alpha$.
- $\text{Valid}(\text{ck}, \text{vk}, m, a, d, \text{vtk})$: Checks $a \stackrel{?}{=} h^m \cdot d^{1/\text{vtk}}$.

Chameleon Hash: SIS

[CHKP10,MP12]

- $\text{KeyGen}(\mathcal{R})$: $\vec{A}_0 \xleftarrow{\$} \mathbb{Z}_q^{\mathcal{R} \times \ell}$, $(\vec{A}_1, \vec{R}_1) \leftarrow \text{GenTrap}^{\mathcal{D}}(1^{\mathcal{R}}, 1^m, q)$.
Defines $\text{ck} = (\vec{A}_0, \vec{A}_1)$ and $\text{tk} = \vec{R}_1$.
- $\text{VKeyGen}(\text{ck})$: Outputs $\text{vk} = \perp$, $\text{vtk} = \perp$
- $\text{CH}(\text{ck}, \text{vk}, \vec{M}; \vec{r})$: $\vec{r} \leftarrow D_{\mathbb{Z}^m, s \cdot \omega(\sqrt{\log \mathcal{R}})}$, $\vec{C} = \vec{A}_0 \vec{M} + \vec{A}_1 \vec{r}$. Returns \vec{C}, \vec{r} .
- $\text{Coll}(\text{tk}, (\vec{M}_0, \vec{r}_0), \vec{M}_1)$: Outputs
 $\vec{r}_1 \leftarrow \text{SampleD}(\vec{R}_1, \vec{A}_1, (\vec{A}_0 \vec{M}_0 + \vec{A}_1 \vec{r}_0) - \vec{A}_0 \vec{M}_1, s)$.
- $\text{Verif}(\text{ck}, \text{vtk}, \vec{M}, \vec{C}, \vec{r})$: $\|\vec{r}\|$ small, and $\vec{C} \stackrel{?}{=} \vec{A}_0 \vec{M} + \vec{A}_1 \vec{r}$.

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- $\text{Encrypt}(\text{pk}, d; r)$: $\mathcal{C} = (u = g_1^r, v = g_2^r, e = h_1^r \cdot d, w = (c_1 c_2^\theta)^r)$, where $\theta = H(\ell, u, v, e)$.
- $\text{Decrypt}(\text{dk}, \mathcal{C})$: If $w \stackrel{?}{=} u^{x_1 + \theta y_1} v^{x_2 + \theta y_2}$, then compute $M = e/u^z$.

SPHF on valid encryption of valid chameleon witness

- $\text{ProjKG}(\mathcal{C}, b)$: Computes the projection keys $\text{hp} = h^\lambda f^\kappa, h_1^\kappa g_1^\mu g_2^\nu (c_1 c_2^\beta)^\theta$.
- $\text{Hash}(\mathcal{C}, \text{hk})$ $H = (\mathcal{C}/g^{m_i})^\lambda \cdot b^{\text{hk}}$.
- $\text{ProjHash}(\mathcal{C}, b, \text{hp})$: The prover will compute $H' = \text{hp}^s \text{hp}^r$.

CCA-2: Cramer Shoup

[CS02]

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CCA-2 ?

- We need an SPHF compatible encryption.
- Only [KV09] is known, and only for approximate SPHF, and is only CCA-1
- However $\text{CCA-1} + \text{S-OTS} \Rightarrow \text{CCA-2}$, and Chameleon Hashes gives S-OTS
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- ✓ Constructions under classical assumptions (DCR, DDH, LWE) in the standard model
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